Let a piecewise linear map $f(x)$ be defined as illustrated in the figure below.

![Graph of a piecewise linear map](image)

In mathematical terms, let $\frac{1}{2} \leq a \leq 1$, and

$$f(x) = \begin{cases} 
\frac{1}{2} + 2(a - \frac{1}{2})x, & 0 \leq x < \frac{1}{2}, \\
\frac{3}{2} - x, & \frac{1}{2} \leq x < \frac{3}{4}, \\
\frac{1}{2} - 2(x - \frac{3}{4}), & \frac{3}{4} \leq x \leq 1.
\end{cases}$$

Consider the dynamical system $x_{t+1} = f(x_t)$.

a) Start with $a = 1/2$ and discuss how the dynamics change when $a$ is increased. Determine whether there is a stable fixed point, stable periodic orbit, or chaos. Is there a critical value for $a$, for which there is a change in dynamical characteristics?

b) Suppose now that $a = 1$. Determine the invariant measure that characterizes the chaotic behaviour, and calculate the Lyapunov exponent $\lambda$. Find a partition that is generating, and calculate the measure entropy from the symbolic dynamics. If you know that $x_t$ is in the interval $[0, \frac{1}{2}]$ at time $t$, how much information do you gain if you learn that also $x_{t+2}$ is in the same interval?

If you use equations or other results from the lectures or lecture notes, make sure to reference them and motivate why they may be used.